The 1st International Conference On Mathematics Education And Technology (ICOMET) 2022
pp. 57-61 (2022)
Penerbit: Universitas Islam Malang

OVERVIEW OF THREE WORLDS OF MATHEMATICS: THINKING SKILLS OF FIRST YEAR STUDENTS IN SOLVING INDEFINITE FORM OF LIMIT PROBLEMS

Endrayana Putut Laksminto Emanuel[1*], Suhartono[2]

[1,2]Mathematics Education Study Program, Faculty of Language and Science, Universitas Wijaya Kusuma Surabaya, Jl. Dukuh Kupang XXV 54, Surabaya, Indonesia

ABSTRACT

This study aims to provide an overview of the change in thinking skills from mathematics in high school to mathematics at university about the limit of indefinite forms. Mathematics in universities is shifting towards mathematical proofs and formal frameworks of thinking. Changes in the way of thinking are reviewed with the framework of three mathematical worlds, namely conceptual-embodied based on perception, action and experiment; proceptual-symbolic based on algebraic computation and manipulation; and axiomatic-formal based on the concept of mathematical definitions and proofs. The descriptive quantitative research method consists of the stages of preparation, data collection, data transcription, data analysis. A total of 20 first year graduate students in mathematics education were given a math problem sheet, containing two numbers regarding the limit of indefinite forms. The results of students work are grouped based on the concept of three worlds. As many as 18 students or 90% are in the proceptual-embodied world, and two students or 10% are in the axiomatic-formal world. No student is in a conceptual-embodied world. From each group, one student was selected to conduct semi-structured interviews as research triangulation. The results of the interview show that students' thinking abilities are still influenced by the knowledge they previously acquired while in high school and further research is needed to improve their thinking skills in accordance with the real world of mathematics in university.

Keywords: thinking skills, mathematics in university, first year graduate student, undefinite form of limit function

1. Introduction

This study aims to provide an overview of the change in thinking skills from mathematics in high school to mathematics at university about the limit of indefinite forms. Mathematics in universities is shifting towards mathematical proofs and formal frameworks of thinking(Ioannou, 2016; Kim et al., 2019; Sfard, 2014). Changes in the way of thinking are reviewed with the framework of three mathematical worlds, namely conceptual-embodied based on perception, action and experiment; proceptual-symbolic based on algebraic computation and manipulation; and axiomatic-formal based on the concept of mathematical definitions and proofs(Tall, 2008). Mathematics in university is different from mathematics in high school(E. P.L. Emanuel, Kirana, & Chamidah, 2021; Endrayana Putut Laksminto Emanuel & Meilantifa, 2022). It deals with individual development in high school and at university. Three things that are fundamentally related to individual development include recognition, repetition, and language where these three things are interconnected to form mathematical thinking skills(Ioannou, 2017; Tall, 2008). However, the recognition and categorization of images and planes lies in geometry and graphics, where the repetition of actions symbolized as conceivable concepts leads to arithmetic and algebra. Each of these construction processes leads further through the use of language to describe, define, and infer relationships, from high school to university level, where theoretical language is used as the basis of formal mathematical theory(Tasara, 2017).

The three worlds of mathematics are built from various advantages, including:

1. The conceptual-embodied world, the conceptually embodied world, based on perception and reflection on the properties of objects, is initially seen and felt in the real world but is then imagined in the mind.
2. The proceptual-symbolic world, the proceptual-symbolic world that grows out of the embodied world through action (such as counting) and is symbolized as something that can be thought of a concept (such as a number) that serves both as a process to be done and concepts to think about (procepts);
3. The axiomatic-formal world, the axiomatic-formal world (based on formal definitions and proofs), which reverses the order of meaning construction from definitions based on known objects to formal concepts based on a collection of theoretical definitions.

Terms such as 'embodied', 'symbolic', 'formal' have all been used in a variety of different ways. Two words that are familiar together in a new way of signaling the need to construct new meanings (such as 'instrumental understanding' and 'relational understanding' or 'concept image' and 'concept definition'). That all thought is embodied, but more specifically for the perceptual representation of concepts. Conceptually embody a geometric figure, such as a triangle consisting of three straight line segments; we imagine a triangle as such an image and allow a particular triangle to act as a prototype to represent an entire class of triangles. We 'see' a specific graphic image that represents a particular or general function(Sudarsana et al., 2019). So that, conceptual embodiments grow increasingly sophisticated as individuals mature in the manner, constructing from object perception, through description, construction and definition, leading to deduction and interpretation. Other materialized geometries followed, such as projective geometries, spherical geometries, and various non-Euclidean geometries, all of which could be given physical embodiments(Khan & Krell, 2019). So that, it is only when systems are axiomed and properties are deduced solely from axioms using set theoretical formal proofs that the cognitive development of geometry shifts completely to formal-axiomatic approach (See Figure 1).

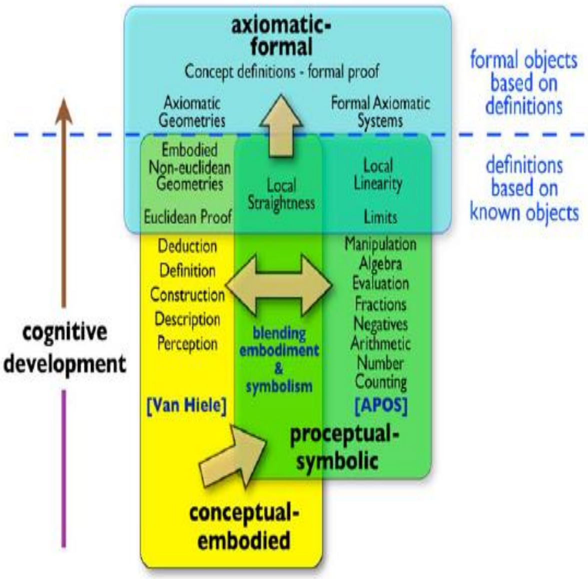

Figure 1. The three worlds of Mathematics

2. Method

The descriptive quantitative research method consists of the stages of preparation, data collection, data transcription, data analysis. Students had given a sheet of paper with a problem. The problems are

1. $\lim\limits_{x\to 1}\frac{x^2-1}{x-1}$
2. $\lim\limits_{x\to 4}\frac{\sqrt{x}-2}{x-4}$

For 10 minutes, they try to solve that problem and thinking hard for that. After 10 minutes, the student’s work were submitted. From their work, I analyze about the data. The conclusion were arise based on their work.

3. Result

A total of 20 first year graduate students in mathematics education were given a math problem sheet, containing two numbers regarding the limit of indefinite forms. The results of students work are grouped based on the concept of three worlds. As many as 18 students or 90% are in the proceptual-embodied world, and two students or 10% are in the axiomatic-formal world. No student is in a conceptual-embodied world.

4. Discussion

Proceptual symbolism refers to the use of symbols that arise from presenting schematic actions, such as counting, which become conceivable concepts, such as numbers. Symbols such as 3+2 or $b^2 - 4ac$ represent either the process to be performed or the conceivable concept generated by the process. For example, the combination of symbols, processes, and concepts built from processes is called a basic principle; A collection of basic concepts with the same output concept is called a concept.

'Axiomatic formalism' refers to Hilbert's formalism which takes us beyond Piaget's formal operations. The main difference from elementary mathematics embodiment and symbolism is that in elementary mathematics, definitions arise from experience with objects whose properties are described and described used as a definition; in formal mathematics, as written in mathematics publication, a formal presentation begins with a collection of theoretical definitions and concludes other properties using formal proofs.

School mathematics is built on the embodiment of physical concepts and actions: playing with shapes; put them in a collection; pointing and counting; share; measuring. Once these operations are practiced and become routine, they can be denoted as numbers and used both as operations or as mental entities on which operations can be performed. As the focus of attention shifts from the embodiment to the manipulation of symbols, mathematical thinking shifts from the embodied to the symbolic (proceptual) world. Throughout school mathematics, embodiment gives special meaning in varying contexts while symbolism. Cognitive development through the three worlds of mathematics(Tall, 2008). Arithmetic and algebra offer the mental world the power of computing(Stewart, Troup, & Plaxco, 2019). The subsequent transition to the formal axiomatic world builds on these experiences of embodiment and symbolism to formulate formal definitions and to prove theorems by means of mathematical proofs. Written formal proof is the final stage of mathematical thinking; it builds on the experience of what theorems may be worth proving and how the proof can be done, often building implicitly on embodied and symbolic experience(E. P.L. Emanuel et al., 2021).

Formal theories based on axioms often lead to structure theorems, which express that axiomatic systems (such as vector spaces) have associated embodiments and symbolism—for example a finite-dimensional vector space is an n-dimensional coordinate system. In this way the theoretical framework turning full circle, building from embodiment and symbolism to formalism,return once again to more sophisticated forms of embodiment and symbolism which, in turn, provide new ways of understanding mathematics that are even more sophisticated.

Calculus is built on three very different worlds of mathematics(Tall, 2008). Calculus in school is a blend of the world of embodiment (drawing graphics) and symbolism (manipulating formulas). The geometric notion of the slope of a graph is often represented by the act of moving a secant through a point on the graph towards a tangent to the point or, more subtly, by zooming in on the graph near the point to see it look like a straight line under high magnification. The latter allows the learner to 'see' the change in the slope of the curve and imagine the slope itself as a changing function. The symbolic aspect allows the slope between two different the points to be calculated numerically or symbolically and the delimiting process is required to obtain the symbolic slope of the tangent as a symbolic derivative. The embodied version has an implicit boundary process in the enlargement process, whereas the symbolic version involves computing an explicit symbolic representation. It is interesting to note that mathematicians, already having conceptions of derivatives, integrals, and so on, had the concept of limit as a confluence before and saw calculus as a logical construct of the concept of limit, hence designing curriculum to build an 'informal' version of the concept of boundaries.

However, beginners may feel more comfortable with the approach embodied through magnification to 'see' the slope function before being introduced to the symbolic techniques for calculating them and the formal language for defining them. No regular calculus study tries to provide insight into what it means to be indistinguishable, but I did in my first lesson on calculus to show some local functions are straight and some aren't. If one can imagine, in the eye's mind, that the graph is locally straight, then as the eye follows the curve from left to right, focusing on the slope of the curve, it is possible to see the changing slope as a function that can be graphed by itself. This brings us right to the principle stated earlier, that the slope can be realized and visualized giving the slope function which is visible but now needs to be calculated either numerically or symbolically. The need for limits arises from the embodiment of calculating the slope function, not the other way around.

Approaches using local linearity, as in Calculus in university, on the other hand, involve a symbolic concept, seeking the best linear approximation to a curve at a point. It involves the concept of an explicit constraint from the start instead of the concept of an implicit constraint that occurs when zooming in to see how steep the curve is in short intervals. Non-differentiation is the absence of a limit, which has no closeness to the idea embodied from a graph that is not zoomed in to see locally straight. A total of 18 students solved the problem of limit of indefinite form by factoring or using differential. This factoring or differentiation activity is a step or procedure taken to solve a problem. These activities are characteristic of the embodied proceptual world. In the embodied proceptual world, problems are solved according to procedures or rules that have been understood by students. Step by step is carried out according to the problem being solved. The completion of the limit of indefinite form begins with identifying the problem, then solving the problem in accordance with the procedures that have been understood previously. The factoring step changes the indefinite form into a certain limit form after simplifying the function first. After these steps are carried out, the limit solution is obtained. The step of differentiation is also performed when it is in the form of an indeterminate limit. After doing the differential to the function whose limit is sought, then a certain limit is obtained. Certain limits can be more easily found a solution. A total of 2 students did proof with formal evidence. This shows that the student uses formal proof to show that the mathematical problem can be solved. The use of formal proofs in solving problems shows that both students are in a formal axiomatic world. Mathematical problems that are solved using formal proofs show that the two students are in a formal axiomatic world. Meanwhile, in the conceptual-embodied world, there are no students. This illustrates that to imagine the problem in question into everyday life can be solved.

5. Conclusion

Most students solve the problem of limit of indefinite form by factoring or using differentials. This factoring or differentiation activity is a step or procedure taken to solve a problem. These activities are characteristic of the embodied proceptual world. In the embodied proceptual world, problems are solved according to procedures or rules that have been understood by students. Step by step is carried out according to the problem being solved. The completion of the limit of indefinite form begins with identifying the problem, then solving the problem in accordance with the procedures that have been understood previously. The factoring step changes the indefinite form into a certain limit form after simplifying the function first. After these steps are carried out, the limit solution is obtained. The step of differentiation is also performed when it is in the form of an indeterminate limit. After doing the differential to the function whose limit is sought, then a certain limit is obtained. Certain limits can be more easily found a solution. A small number of students do proof with formal evidence. The use of formal proofs in solving problems shows that both students are in a formal axiomatic world. This shows that the number of students in the formal axiomatic world is still small. Meanwhile, in the conceptual-embodied world, there are no students. This illustrates that the majority of first year students are still in the proceptual embodied world, a small proportion of students are in the formal axiomatic world. The results show that students' thinking abilities are still influenced by the knowledge they previously acquired while in high school and further research is needed to improve their thinking skills in accordance with the real world of mathematics in university.

6. References

Emanuel, E. P.L., Kirana, A., & Chamidah, A. (2021). Enhancing students' ability to solve word problems in Mathematics. Journal of Physics: Conference Series, 1832(1). https://doi.org/10.1088/1742-6596/1832/1/012056

Emanuel, Endrayana Putut Laksminto, & Meilantifa. (2022). Dimanakah Nilai Ekstrim Fungsi Kuadrat Ditinjau dari Lensa Commognitive ? BRILIANT Jurnal Riset Dan Konseptual, 7(54), 269–279.

Ioannou, M. (2016). Commognitive Analysis of Undergraduate Mathematics Students' Responses in Proving Subgroup's Non-Emptiness. Opening up Mathematics Education Research (Proceedings of the 39th Annual Conference of the Mathematics Education Research Group of Australasia), 344–351.

Ioannou, M. (2017). Investigating the discursive shift in the learning of Group Theory: Analysis of some interdiscursive commognitive conflicts. Retrieved from https://hal.archives-ouvertes.fr/hal-01941352/document

Khan, S., & Krell, M. (2019). Scientific Reasoning Competencies: a Case of Preservice Teacher Education. Canadian Journal of Science, Mathematics and Technology Education. https://doi.org/10.1007/s42330-019-00063-9

Kim, D.-J., Choi, S., Lim, W., Thoma, A., Nardi, E., Viirman, O., … Sfard, A. (2019). Discourses of Functions – University Mathematics Teaching Through a Commognitive Lens. Educational Studies in Mathematics, 8(2), 423–430. https://doi.org/10.1007/s10649-015-9676-1

Sfard, A. (2014). University mathematics as a discourse - why, how, and what for? Research in Mathematics Education, 16(2), 199–203. https://doi.org/10.1080/14794802.2014.918339

Stewart, S., Troup, J., & Plaxco, D. (2019). Reflection on teaching linear algebra: examining one instructor's movements between the three worlds of mathematical thinking. ZDM - Mathematics Education, 51(7), 1253–1266. https://doi.org/10.1007/s11858-019-01086-0

Sudarsana, I. K., Nakayanti, A. R., Sapta, A., Haimah, Satria, E., Saddhono, K., … Mursalin, M. (2019). Technology Application in Education and Learning Process. Journal of Physics: Conference Series, 1363(1). https://doi.org/10.1088/1742-6596/1363/1/012061

Tall, D. O. (2008). The transition to formal thinking in mathematics. Mathematics Education Research Journal. https://doi.org/10.1007/BF03217474

Tasara, I. (2017). Commognitive analysis of a teacher's mathematical discourse on the derivative. Proceedings of the British Society for Research into Learning Mathematics.